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Our approximations are not useful for hard spheres since perturbations are then larger and volume exclusion effects more complicated. The analogous formulation in terms of higher order distributions would serve better here, and is being worked out.

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THE VISCOSITY OF KRYPTON BETWEEN 25° AND 75°C AND AT PRESSURES UP TO 2000 ATM

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Synopsis

The viscosity of krypton has been measured with a transpiration method at 25°, 50°, nd 75°C, at pressures up to 2050 atmospheres, and at 125°C, at pressures between 300 and 1900 atmospheres. The results agree within 1% with the values calculated om a formula in reduced quantities for the noble gases, published previously.

§ 1. *Introduction.* With the standard transpiration method of the laboratory, the viscosity of krypton was measured at temperatures of 25°, 50°, 75°, nd 125°C. The pressure range was between 7 and 2052 atmospheres, except t 125°C, where measurements were made only between 1294 and 1880 tmospheres. The density corresponding to each temperature and pressure as calculated using the compressibility isotherms of krypton, measured in his laboratory[1]).

§ 2. *Results.* The results of the measurements are given in tables I to IV. The pressure p is expressed in normal atmospheres, the density ρ both in amagat units and in g cm^{-3}, and the coefficient of viscosity η in g cm^{-1} s^{-1}. The viscosity pressure isotherms shown in fig. 1 have the same general eatures as the corresponding curves for nitrogen[2)3)], hydrogen[4)5)], leuterium[4)5)], argon[6)] and neon[7)]. In contradistinction with these gases, owever, the viscosity density isotherms of krypton show an intersection t high density (fig. 2).

It was possible, by means of least squares, to fit to the isotherms of 25°, 0°, and 75°C polynomials of the form

$$\eta \cdot 10^6 = A + B\rho + C\rho^2 + D\rho^3 \qquad (1)$$

or the density range 6–330 amagat. The coefficients of the polynomials re given in table V.

Polynomials of the form

$$\eta \cdot 10^6 = A + B\rho + C\rho^2 + D\rho^3 + E\rho^4 + F\rho^5 + G\rho^6 \qquad (2)$$

TABLE I

The viscosity of krypton at 25°C

p atm	ρ am	ρ g cm^{-3}	$\eta \cdot 10^6$ (exp) g cm^{-1}s^{-1}	$\eta \cdot 10^6$ calc. from (1)	$\eta \cdot 10^6$ calc. from (2)	$\eta \cdot 10^6$ calc. from (3)
7.00	6.49	0.0243	257.0	256.0	256.9	252.5
8.33	7.74	0.0290	257.0	256.5	257.4	253.2
9.65	8.99	0.0337	257.4	257.1	257.7	253.8
19.51	18.58	0.0695	261.3	261.5	261.4	258.9
23.38	22.44	0.0841	263.4	263.4	263.0	261.0
40.28	40.15	0.1505	272.2	272.8	271.9	271.5
49.07	49.88	0.1870	278.0	278.5	277.6	277.8
57.72	59.85	0.2244	284.2	284.8	284.0	284.5
66.17	69.95	0.2623	291.4	291.5	291.0	291.8
75.86	81.99	0.3074	299.6	300.2	300.0	301.1
86.08	95.19	0.3569	310.4	310.4	310.6	311.9
94.47	106.4	0.3988	319.6	319.8	320.3	321.7
109.4	127.0	0.4762	339.6	338.8	339.6	341.4
125.7	150.4	0.5639	364.3	363.1	364.0	366.3
134.9	163.6	0.6134	379.3	378.2	379.1	381.6
146.5	180.4	0.6765	399.3	399.0	399.6	402.5
159.7	199.1	0.7463	424.4	424.1	424.4	427.6
180.1	226.3	0.8485	464.7	465.1	464.7	468.2
199.8	250.5	0.9390	504.7	505.8	504.8	508.0
224.0	276.8	1.0378	554.3	555.2	553.9	556.1
248.6	299.9	1.1244	602.8	603.0	602.0	602.4
284.1	327.8	1.2290	667.7	666.7	667.1	663.6
332.6	358.2	1.3429	747.8		748.1	
383.6	383.6	1.4382	823.0		825.2	
484.7	421.5	1.5803	959.8		959.5	
604.4	454.4	1.7036	1101		1100	
723.5	479.8	1.7987	1225		1227	
908.5	510.6	1.9144	1411		1411	
1140.2	540.6	2.0266	1630		1628	
1446.0	571.5	2.1425	1901		1905	
1657.2	589.2	2.2090	2095		2094	
1798.1	599.8	2.2488	2221		2220	

were calculated for the entire density region 6–600 amagat. The coefficients are given in table VI. The values for η calculated from (1) and (2), for the densities at which the measurements were performed, are listed in tables I to III (fifth and sixth columns respectively).

§ 3. *Corresponding states.* In a previous paper[7]) a formula in reduced quantities is given, which allows calculation of the viscosity for the noble gases:

$$\frac{\eta^*}{\eta_0^*} = \alpha(\rho^*) + \beta(\rho^*) \log T^* + \gamma(\rho^*) \log^2 T^* + \delta(\rho^*) \log^3 T^*, \qquad (3)$$

TABLE II

The viscosity of krypton at 50°C

p atm	ρ am	ρ g cm^{-3}	$\eta \cdot 10^6$ (exp) g cm^{-1} s^{-1}	$\eta \cdot 10^6$ calc. from (1)	$\eta \cdot 10^6$ calc. from (2)	$\eta \cdot 10^6$ calc. from (3)
7.53	6.42	0.0241	274.7	274.9	275.7	270.6
8.99	7.69	0.0288	275.9	275.5	276.1	271.2
10.50	9.00	0.0337	276.7	276.1	276.6	271.9
20.91	18.21	0.0683	280.7	280.5	280.3	276.8
25.54	22.41	0.0840	282.8	282.6	282.2	279.1
44.43	40.18	0.1506	291.9	292.4	291.5	289.6
54.34	49.92	0.1871	297.8	298.2	297.4	295.8
64.28	59.98	0.2249	304.2	304.6	304.0	302.6
73.97	70.05	0.2626	311.4	311.5	311.2	309.8
84.27	81.02	0.3038	319.2	319.5	319.5	318.2
97.20	95.18	0.3568	330.4	330.6	331.0	329.8
107.3	106.5	0.3993	340.3	340.1	340.8	339.7
127.1	129.1	0.4839	360.9	361.1	362.1	361.2
145.5	150.3	0.5634	383.9	383.2	384.1	383.9
157.4	164.0	0.6147	399.6	398.8	399.5	399.8
172.4	181.0	0.6784	419.8	419.7	420.0	421.1
190.5	200.9	0.7530	446.8	446.4	446.2	448.2
215.3	226.4	0.8487	484.7	484.4	483.3	486.7
241.5	251.0	0.9411	525.5	525.2	523.7	528.0
271.5	276.2	1.0354	569.6	571.3	570.0	574.7
304.3	300.1	1.1252	618.7	619.7	619.3	623.7
349.9	328.2	1.2305	683.5	682.2	684.7	686.9
410.4	358.5	1.3439	763.0		765.2	
473.4	384.0	1.4397	843.5		842.9	
595.2	422.1	1.5824	979.3		978.2	
742.4	456.4	1.7110	1128		1125	
870.2	479.9	1.7993	1241		1243	
1072.7	510.0	1.9120	1420		1420	
1329.9	540.1	2.0249	1633		1633	
1651.9	570.1	2.1374	1893		1893	
2052.0	600.1	2.2496	2210		2210	

where

$$T^* = \frac{k}{\varepsilon} T, \rho^* = \frac{\sigma^3}{m} \rho, \eta^* = \frac{\sigma^2}{\sqrt{m\varepsilon}}, \eta_0^* = \frac{5}{16\sqrt{\pi}} \cdot \frac{\sqrt{T^*}}{\Omega^{(2,2)*}(T^*)},$$

and σ are the two constants for the Lennard-Jones intermolecular 6–12 potential field, m is the mass per molecule, ρ is the density in g/cm^3, $\Omega^{(2,2)*}(T^*)$ is a reduced collision integral.

The coefficients for the range: ρ^* from 0 to 0.45, T^* from 1.2 to 30, were calculated from viscosity measurements of helium, neon, argon, and xenon.

TABLE III

The viscosity of krypton at 75°C

p atm	ρ am	ρ g cm^{-3}	$\eta \cdot 10^6$ (exp) g cm^{-1} s^{-1}	$\eta \cdot 10^6$ calc. from (1)	$\eta \cdot 10^6$ calc. from (2)	$\eta \cdot 10^6$ calc. from (3)
8.24	6.51	0.0244	294.7	293.6	294.6	287.9
11.35	9.00	0.0338	295.4	294.7	295.5	289.2
22.26	17.88	0.0670	298.9	299.1	299.1	293.9
27.81	22.49	0.0843	301.3	301.5	301.2	296.4
48.53	40.18	0.1506	310.9	311.2	310.5	306.8
59.51	49.89	0.1870	316.4	317.1	316.3	313.0
70.68	59.96	0.2248	323.2	323.5	322.8	319.7
81.62	70.02	0.2625	330.1	330.4	329.9	326.9
93.47	81.11	0.3041	337.6	338.4	338.3	335.3
108.3	95.21	0.3570	349.3	349.5	349.6	346.7
120.4	106.9	0.4008	360.0	359.3	359.7	356.9
143.2	129.0	0.4838	380.2	379.8	380.5	378.0
165.2	150.3	0.5636	402.8	401.9	402.7	400.7
179.3	163.8	0.6142	418.7	417.3	417.9	416.4
197.7	181.0	0.6785	438.1	438.3	438.7	437.9
219.7	200.8	0.7527	465.0	464.8	464.8	465.0
249.6	225.9	0.8469	502.4	502.2	501.7	503.1
282.0	250.7	0.9397	543.1	543.4	542.4	545.0
320.7	277.0	1.0383	590.7	592.0	591.1	594.6
358.3	299.4	1.1224	636.5	637.8	637.6	641.2
415.2	328.4	1.2310	704.7	703.1	705.0	707.7
486.1	358.0	1.3423	782.1		784.0	
564.2	384.8	1.4426	866.7		865.2	
704.5	422.5	1.5838	1003		998.4	
1013.8	480.0	1.7994	1256		1260	
1237.8	510.0	1.9120	1432		1434	
1518.3	540.0	2.0246	1645		1642	
1865.9	570.0	2.1370	1896		1895	
2023.9	581.8	2.1811	2008		2009	

TABLE IV

The viscosity of krypton at 125°C

p atm	ρ am	ρ g cm^{-3}	$\eta \cdot 10^6$ (exp) g cm^{-1} s^{-1}
1294.2	480.0	1.7995	1296
1555.2	509.5	1.9102	1461
1880.4	539.5	2.0228	1664

They are given by (see reference 7)

$$\alpha(\rho^*) = 0.980 + 2.140\rho^* + 8.171\rho^{*2} - 0.563\rho^{*3}$$
$$\beta(\rho^*) = -3.170\rho^* - 25.720\rho^{*2} + 27.665\rho^{*3}$$
$$\gamma(\rho^*) = 1.278\rho^* + 30.191\rho^{*2} - 43.189\rho^{*3}$$
$$\delta(\rho^*) = -0.120\rho^* - 10.388\rho^{*2} + 16.549\rho^{*3}$$

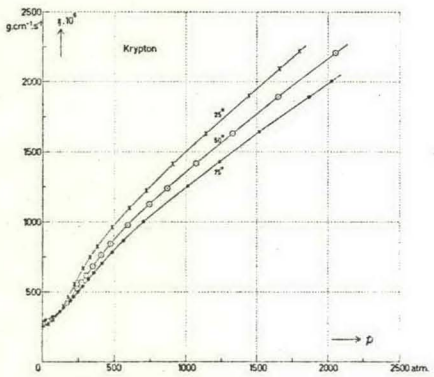


Fig. 1. Viscosity of krypton as function of pressure.

TABEL V

Coefficients of the polynomial $\eta \cdot 10^6 = A + B\rho + C\rho^2 + D\rho^3$ krypton, range 6–330 am. (η in g cm^{-1} s^{-1}; ρ in am.)

	25°C	50°C	75°C
A	253.17	272.01	290.56
B	0.41865	0.43638	0.44933
$C \cdot 10^3$	1.6547	1.6340	1.4992
$D \cdot 10^6$	2.7957	2.5712	2.9188

TABLE VI

Coefficients of the polynomial $\eta \cdot 10^6 = A + B\rho + C\rho^2 + D\rho^3 + E\rho^4 + F\rho^5 + G\rho^6$. krypton, range 6–600 am. (η in g cm^{-1} s^{-1}; ρ in am.)

	25°C	50°C	75°C
A	255.01	273.68	292.47
B	0.26227	0.28421	0.30075
$C \cdot 10^3$	4.56735	4.64178	4.24199
$D \cdot 10^6$	− 18.4219	− 19.9537	− 17.0204
$E \cdot 10^9$	+ 73.9157	+ 77.4699	+ 68.4218
$F \cdot 10^{12}$	− 130.867	− 130.328	− 117.155
$G \cdot 10^{15}$	+ 101.6153	+ 96.4190	+ 88.3948

The η values of krypton, calculated from (3), are given in the last columns of tables I to III. The constants are taken from Whalley and Schneider[8]: $\varepsilon/k = 166.67$°K, $\sigma = 3.681$ Å. It can be seen that, except at low densities,

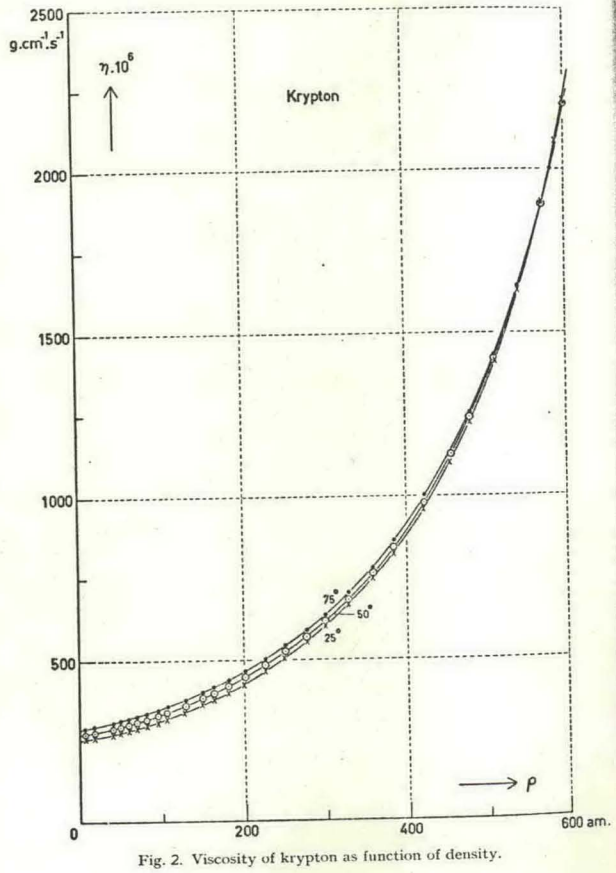


Fig. 2. Viscosity of krypton as function of density.

these values agree with the experimental data within the accuracy of 1% claimed in reference 7. The deviation at low density shall be discussed in a next paper.

§ 4. *Comparison with measurements of other authors.* There are only few viscosity measurements on krypton available in the literature. In the low density region there are data of Nasini and Rossi[9]), Rankine[10]), and Clifton[11]). A survey is given in table XI of reference 7.

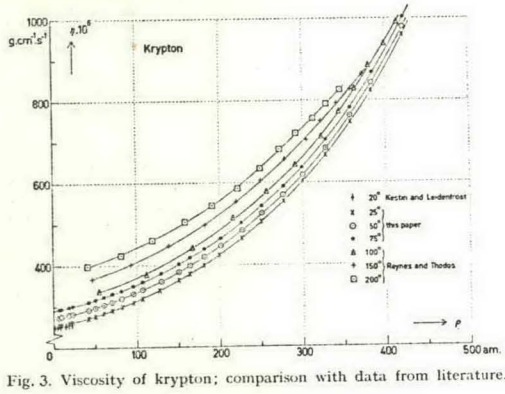


Fig. 3. Viscosity of krypton; comparison with data from literature.

At higher densities there are data from Kestin and Leidenfrost[12]) at 20°C at densities up to 21 am, and very recently from Reynes and Thodos[13]) at 100°C, 150°C, and 200°C at densities between 40 and 414 am. In fig. 3 these values are plotted versus density together with our own data. It will be noted that the viscosity isotherms of Reynes and Thodos at 150° and 200°C appear to be irregular. This behaviour is confirmed when a comparison is made with the values calculated from equation (3). Deviations up to 4% are found for the 150° and 200°C isotherm, but it is worth noting that the (smoothed) 100° isotherm shows deviations of less than 1%.

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THE MAXWELL EQUATIONS: NONRELATIVISTIC MULTIPOLE EXPANSION TO ALL ORDERS

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Synopsis

From the microscopic equations for the electromagnetic field in the presence of int sources equations are derived for the electromagnetic field in the presence of able collections of point sources such as atoms, ions or molecules. These "atomic ld equations" contain electric and magnetic polarizations. The polarizations are en here as expansions to all orders in terms of the nonrelativistic electric and magnetic ltipoles of the atoms.

§ 1. *Introduction.* The microscopic equations for the electromagnetic ld $(\boldsymbol{e}, \boldsymbol{b})$, produced at a time t and a position $\boldsymbol{R}$ by a collection of point arges $e_j (j = 1, 2, \ldots)$ at positions $\boldsymbol{R}_j(t)$ and with velocities $\dot{\boldsymbol{R}}_j(t)$, can be itten as

$$\nabla \cdot \boldsymbol{e} = \sum_j e_j \, \delta(\boldsymbol{R}_j - \boldsymbol{R}), \tag{1}$$

$$-\partial_0 \boldsymbol{e} + \nabla \wedge \boldsymbol{b} = \sum_j e_j (\dot{\boldsymbol{R}}_j / c) \, \delta(\boldsymbol{R}_j - \boldsymbol{R}), \tag{2}$$

$$\nabla \cdot \boldsymbol{b} = 0, \tag{3}$$

$$\partial_0 \boldsymbol{b} + \nabla \wedge \boldsymbol{e} = 0. \tag{4}$$

Here ∂_0 and ∇ indicate differentiations with respect to time and space ordinates ct and $\boldsymbol{R}$ (where c is the velocity of light).

If the point charges are grouped into stable atoms, molecules or ions eferred to as "atoms" here), the problem arises as to how the equations)-(4) can be rewritten with the help of suitably defined "atomic" para- eters. This aim can be achieved by grouping the source terms in (1) and into sums of terms each of which is itself a sum over the constituent clei and electrons of a single atom. Subsequently one can develop each m into a series expansion in terms of atomic electromagnetic multipoles. In this article a treatment is followed originally carried out by Mazur and ijboer[1]) up to second order. However, here no statistical averaging will performed before the multipole expansion is worked out. In this way tomic field equations" will be obtained in which the electric and magnetic